1. (8 points) Let $f(x) = 1 + 2x - 3x^2$.

(a) Simplify the following expression far enough so that plugging in $h = 0$ would be allowed:

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{[1 + 2(x+h) - 3(x+h)^2] - [1 + 2x - 3x^2]}{h}$$

$$= \frac{\cancel{1} + \cancel{2x} + 2h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{1} - \cancel{2x} + \cancel{3x^2}}{h}$$

$$= \frac{2h - 6xh - 3h^2}{h} = \boxed{2 - 6x - 3h}$$

(b) Simplify the following expression far enough so that plugging in $a = 0$ would be allowed:

$$\frac{f(2a) - f(a)}{a}$$

$$\frac{[1 + 4a - 12a^2] - [1 + 2a - 3a^2]}{a}$$

$$= \frac{\cancel{1} + 4a - 12a^2 - \cancel{1} - 2a + 3a^2}{a}$$

$$= \frac{2a - 9a^2}{a} = \boxed{2 - 9a}$$

4. (10 points) Let $f(x) = 3 - x^2$ and $g(x) = \begin{cases} 6x^2 & \text{, if } x < 2 \\ x + 10 & \text{, if } x \geq 2 \end{cases}$.

a) Evaluate and simplify $\dfrac{f(x+a) - f(x)}{a}$. (Simplify as much as possible)

$$= \frac{[3-(x+a)^2] - [3-x^2]}{a} = \frac{3-(x^2+2ax+a^2)-3+x^2}{a}$$

$$= \frac{\cancel{3} - \cancel{x^2} - 2ax - a^2 - \cancel{3} + \cancel{x^2}}{a} = \frac{-2ax - a^2}{a} = \boxed{-2x - a}$$

b) Give a multipart formula for the composition $f(g(x))$.

$$\xrightarrow{x<2} 6x^2 \xrightarrow{f(6x^2)} 3 - (6x^2)^2 = 3 - 36x^4$$

$$\times \quad \xrightarrow{x \geq 2} x+10 \xrightarrow{f(x+10)} 3 - (x+10)^2 \cancel{=3} = ($$

$$\boxed{f(g(x)) = \begin{cases} 3 - 36x^4, & \text{if } x < 2 \\ 3 - (x+10)^2, & \text{if } x \geq 2 \end{cases}}$$

c) Give all values of x that satisfy $g(x) = 6$.

$6 = 6x^2, \quad x < 2$

$1 = x^2$

$x = \pm 1$

both in domain

or

$6 = x + 10 \quad , \text{ if } x \geq 2$

$x \cancel{= -4}$

not in domain

$$\boxed{x = -1, \quad x = +1}$$

4. (12 points) Let $g(x) = x + 7$, $h(x) = x^2 + 1$, and $f(x) = \begin{cases} x^2 + 2 & , \text{ if } x \geq 2 \\ 2x - 1 & , \text{ if } x < 2 \end{cases}$.

(a) The function $h(g(x))$ is quadratic.
Give the formula for $h(g(x))$ and find the x and y coordinates of the vertex.

$$h(g(x)) = h(x+7) = (x+7)^2 + 1$$

$$\boxed{h(g(x)) = x^2 + 14x + 50}$$

Vertex: $x = -\frac{b}{2a} = -\frac{14}{2} = \boxed{-7}$

$y = (-7)^2 + 14(-7) + 50 = \boxed{1}$

(b) Find all solutions for x in the following equation:

$$f(x) = 11$$

$x^2 + 2 = 11$, $x \geq 2$ or $2x - 1 = 11$ $x < 2$

$x^2 = 9$ in domain not in domain

$\boxed{x = +3}$ or $\cancel{x = -3}$

$2x = 12$

$\cancel{x = 6}$ not in domain

$$\boxed{x = 3 \text{ is the only sol'n}}$$

(c) Write the multipart rule for $f(g(x))$.

$$\begin{cases} (x+7)^2 + 2 & , \text{ if } x+7 \geq 2 \\ 2(x+7) - 1 & , \text{ if } x+7 < 2 \end{cases}$$

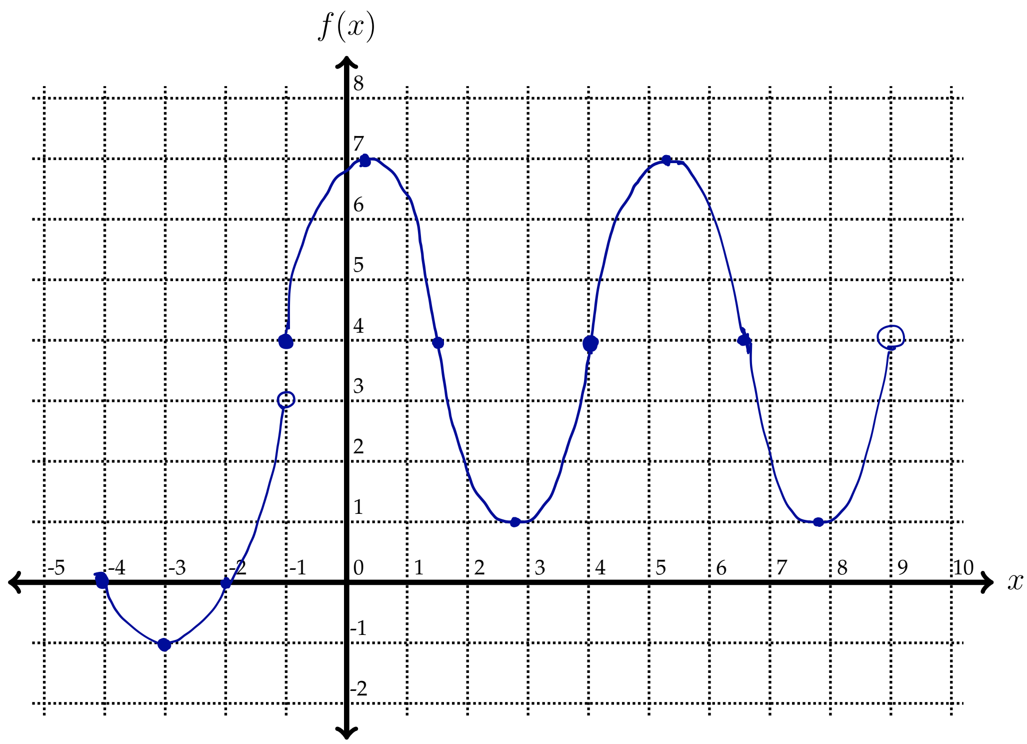
$$\boxed{= \begin{cases} (x+7)^2 + 2 & , \text{ if } x \geq -5 \\ 2x + 13 & , \text{ if } x < -5 \end{cases}}$$

6. Consider the following multipart function:

$\rightarrow (x+3)^2 - 1$

$$f(x) = \begin{cases} x^2 + 6x + 8 & \text{if } -4 \le x < -1 \\ 3\sin\left(\frac{2\pi}{5}(x+1)\right) + 4 & \text{if } -1 \le x < 9 \end{cases}$$

(a) **[6 points]** Sketch a graph of $f(x)$. Label your graph clearly.



(b) **[7 points]** Find *all solutions* to the equation $f(x) = 2$.

First piece:

$2 = x^2 + 6x + 8$

$0 = x^2 + 6x + 6$

$x = \frac{-6 \pm \sqrt{36-24}}{2}$

$x = \frac{-6+\sqrt{12}}{2}$ } yup

$\underbrace{x = \frac{-6-\sqrt{12}}{2}}_{\text{not in domain } -4 \le x < -1}$

Second piece:

$2 = 3\sin\left(\frac{2\pi}{5}(x+1)\right) + 4$

$\frac{-2}{3} = \sin\left(\frac{2\pi}{5}(x+1)\right)$

$\sin^{-1}\left(\frac{-2}{3}\right) = \frac{2\pi}{5}(x+1)$

$x = \frac{5}{2\pi}\left(\sin^{-1}\left(\frac{-2}{3}\right)\right) - 1 \approx \overbrace{-1.58069}^{\text{principal sol'n}}$

Symmetry solution

$= 2C + \frac{B}{2} - p = 2.08069$

List of solutions:

$\vdots$

-1.58069

$\left.\begin{matrix} 2.08069 \\ 3.41931 \\ 7.08069 \\ 8.41931 \end{matrix}\right\}$ In domain $-1 \le x < 9$

12.08069

$\vdots$

$$\boxed{\text{So overall: } \frac{-6+\sqrt{12}}{2},\ 2.08069,\ 3.41931,\ 7.08069,\ \text{and}\ 8.41931}$$